



THE EFFECT OF CONGESTION ON THE ESTIMATION OF THE RATE OF PROGRESS AND REGRESSION OF DECISION-MAKING UNITS IN DATA ENVELOPMENT ANALYSIS

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Abstract

DEA is a technique for measuring the efficiency and performance of decision-making units (DMUs) with multiple inputs and outputs. The term overload in the literature DEA refers to the overuse of the source, which reduces the output and thus the efficiency. This problem changes over time, and the progress and rate of decline of the units can be measured by these changes. In this paper, Malmquist productivity index and congestion are used to measure the progress and regression of units over time. The proposed model is able to change the congestion changes in addition to calculating the efficiency and technology changes by changing the relative distances in the Malmquist productivity index. The criterion for congestion is the fraction $(e_o^i(j))/c_o^i(j)$, and the lower it is, the lower the congestion to be evaluated, and if it is 1, there is no congestion. Thus, by applying this criterion in the structure of the Malmquist productivity index, two categories of distances, performances, and congestion are used in the calculation of progress and regression of unity.

Keywords: Congestion, DEA(Data Envelopment Analysis), Progress, Regression

Introduction

DEA is a non-parametric efficiency evaluation method used to calculate the relative efficiency and evaluate the performance of a group of DMUs. Farrell [18] first determined efficiency using a nonparametric method. Charnes et al [6] introduced the CCR model and Banker et al [3] introduced the BCC model, which are the most practical models.

Stan Malmquist [30], a Swedish economist, introduced the Malmquist productivity index as an index of living standards. Caves et al. [5] used Malmquist's productivity index for the first time in production theory. Farr et al. [14] studied productivity growth, technical progress and efficiency changes in industrialized countries. Balf et al. [2] proposed a method for calculating the Malmquist productivity index.

Maudos et al. [31] studied total productivity and human capital. Tulkens and Vanden Eeckaut [41] were devoted to the nonparametric efficiency of productivity. Chen [7] used the non-radial Malmquist productivity index for Chinese industry. Chen and Ali [8] calculated the Malmquist productivity index using DEA Jesus et al. [24] calculated the global Malmquist productivity index with a new perspective. Since the evaluation of decision units using a common set of

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weights (CSW) is of great importance, Kao [26] proposed the Malmquist productivity index based on CSW. Hosseinzadeh Lotfi et al. [20] estimated the random productivity index of Malmquist. Ali Emrouznejad et al. [12] calculated the overall productivity index using fuzzy data. Beijia Huang et al. [21] studied China's enzh security from 2008 to 2017 based on the fuzzy model and Malmquist productivity index. Ghayeghran Sarab et al. [29] They propose an approach to maintain unit efficiency (cost efficiency) under any fluctuation in input costs.

The first article that studied congestion was an article presented by Fare and Svensson [15] in which they examined the congestion of production factors. Then, Fare and Grosskopf [16] studied the measurement of congestion in production. Fare et al. [17] developed Data Envelopment Analysis to calculate the effects of congestion. Another approach studied by Cooper et al. [9] is a covariate-based method that calculates the congestion effect as the difference between observed and expected values. Using ensemble models, Cooper et al. [10] extended the integrated mass model to determine the congestion effect. Jahanshahloo and Khodabakhshi [22] introduced an input relaxation model to improve the results and calculated the input congestion based on the proposed model.

In addition to the above, other studies have been carried out separately by Wei and Yan [42] and Tone and Sahoo [40] that investigated the impact of compression in terms of adverse inputs. Sueyoshi and Sekitani [38] presented an approach capable of evaluating congestion with multiple solutions. Noura et al. [35] proposed a new method for congestion calculation. Khodabakhshi [27] and Asgharian et al. [1] proposed a method to determine congestion in random conditions.

Mehdilozad et al. [32] proposed a method for congestion detection. Mollaeian and Rostami Mal Khalife [33] investigated the presence of congestion in supply chain products.

Based on the method of Noura, Ebrahimzade Adimi et al. [11] proposed a dense cloud plane to separate dense units from other units, while Navidi et al. [34] proposed a density identification method without calculation. Shadab et al [36] proposed the concept of anchor point and developed a corresponding congestion identification method similar to Noura's method.

Sueyoshi and Goto [39] and Fang [13] investigated congestion by including undesirable outputs and proposed congestion detection methods. Hajaji et al [19] investigated congestion considering undesirable outputs in a multi-stage manufacturing process. Sharma and Yu [37] proposed a method for density measurement in a multi-stage manufacturing process based on the FGL method. Khoyini et al.[28] proposed a method based on covariates to better distinguish strong density from weak density. Karimi et al. [25] bIdentificated of congestion by means of integer-valued data envelopment analysis .

In production technology, the congestion stage is one of the stages that reduces output, although it may face excessive consumption of inputs. This concept was seen in efficiency based on DEA concepts, but it was not seen in the concept of productivity (progress and regress) over time.

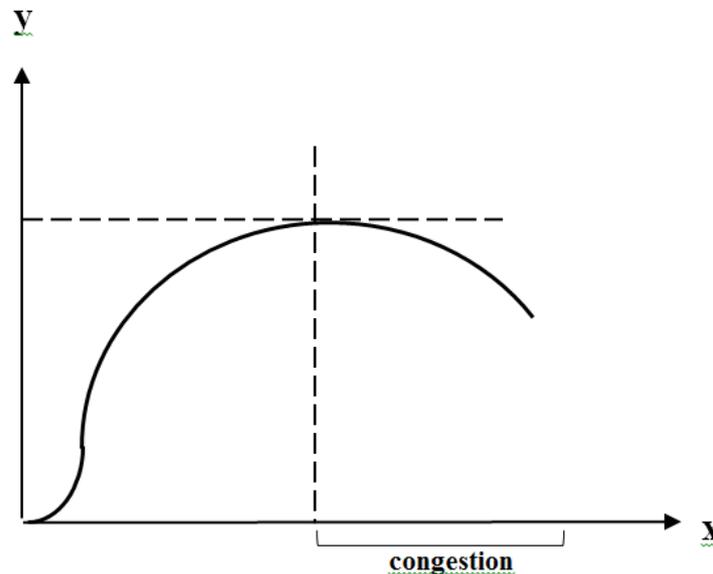


Fig 1-Production function diagram

1. the data envelopment analysis

The production function is a function that gives the maximum output for any combination of inputs. This function is very important in microeconomics because it helps to judge whether a decision unit is working well (is efficient) or not. In most cases, the production function is not available due to the complexity of the production process, changes in production technology, and the large number of production functions. Therefore, an approximation of the production function must be available. Parametric methods such as minimizing the sum of absolute deviations are used to estimate the production function. The method DEA does not aim to calculate this function as a non-parametric method, but aims at all producible activities that include two categories of input-output data. This set can be prepared by adopting a set of principles such as the inclusion of observations, feasibility, etc., whose inviolable limit can be used as an estimate of the production function [23]. The basic models of DEA are CCR model and BCC model [31,32].

CCR model as input:

$$\begin{aligned}
 & \min \theta \\
 \text{s.t. } & \sum_{j=1}^n \lambda_j x_j \leq \theta x_0, \\
 & \sum_{j=1}^n \lambda_j y_j \geq y_0, \\
 & \lambda_j \geq 0 \quad , j = 1, \dots, n.
 \end{aligned}
 \tag{1}$$

CCR model as an output:

$$\begin{aligned}
 & \max \phi \\
 \text{s.t. } & \sum_{j=1}^n \lambda_j x_j \leq x_0, \\
 & \sum_{j=1}^n \lambda_j y_j \geq \phi y_0, \\
 & \lambda_j \geq 0 \quad , j = 1, \dots, n.
 \end{aligned}
 \tag{2}$$

BCC model as an input:

$$\begin{aligned}
 & \min \theta \\
 \text{s.t. } & \sum_{j=1}^n \lambda_j x_j \leq \theta x_0, \\
 & \sum_{j=1}^n \lambda_j y_j \geq y_0, \\
 & \sum_{j=1}^n \lambda_j = 1, \\
 & \lambda_j \geq 0 \quad , j = 1, \dots, n.
 \end{aligned}
 \tag{3}$$

BCC model as an output:

$$\max \phi$$

$$\begin{aligned}
& \text{s.t. } \sum_{j=1}^n \lambda_j x_j \leq x_0, \\
& \sum_{j=1}^n \lambda_j y_j \geq \varphi y_0, \\
& \sum_{j=1}^n \lambda_j = 1, \\
& \lambda_j \geq 0 \quad , j = 1, \dots, n.
\end{aligned} \tag{4}$$

2.the Malmquist productivity index

The production function changes over time because the factors that affect it are time-dependent, which is called technological change. Regardless of the changes that determine it, a given production unit may have functional changes whose source is the internal factors within the unit, leading to changes in the efficiency of that unit over time. These two changes could be integrated into the Malmquist productivity index. For example, a farmer achieves different results at two points in time, which could be due to two factors: technological changes, such as climate changes, or efficiency changes, such as increasing experience or changing personal skills.

The Malmquist productivity index is one of the indexes in economic analysis used to study overall productivity. This index divides productivity into two parts, efficiency change and technology change, so non-parametric programming models can be used to calculate it. The decision units in Data Envelopment Analysis are on the efficiency frontier with the highest output level and the lowest input level. Fare et al. [14] proposed the Malmquist productivity index as the geometric mean of the two Malmquist productivity indices of Caves et al. defined by the distance function $D(\cdot)$. The Malmquist productivity index is the average of the two Malmquist productivity indices. Ques et al. assumed that $D_k(K)=1$, the production technology is efficient with respect to k , and its distance function does not indicate inefficiency.

Fare et al. divided this index into the two factors of efficiency and technological changes. This index is calculated with relative distance functions. Relative distance functions are obtained by the technical efficiency of Data Envelopment Analysis models; therefore, the Malmquist productivity index based on maximization between t and $t + 1$ is written as follows:

$$M_0^{t+1} = \frac{E_0^t(x^{t+1}, y^{t+1})}{E_0^t(x^t, y^t)}. \tag{5}$$

Similarly, the Malmquist productivity index based on maximization within the interval t and $t + 1$ with respect to the efficiency frontier at time and $t + 1$ is as follows:

$$M_o^{t+1} = \frac{E_o^{t+1}(x^{t+1}, y^{t+1})}{E_o^{t+1}(x^t, y^t)}. \quad (6)$$

The above two Malmquist indices are equivalent, and the Malmquist productivity index is the geometric mean of the two productivity indices calculated as follows:

$$M_o = \left[\left(\frac{E_o^t(x^{t+1}, y^{t+1})}{E_o^t(x^t, y^t)} \right) \left(\frac{E_o^{t+1}(x^{t+1}, y^{t+1})}{E_o^{t+1}(x^t, y^t)} \right) \right]^{\frac{1}{2}}. \quad (7)$$

The Malmquist index is divided into efficiency and technological change:

$$M_o = \left(\frac{E_o^{t+1}(x^{t+1}, y^{t+1})}{E_o^t(x^t, y^t)} \right) \left[\left(\frac{E_o^t(x^{t+1}, y^{t+1})}{E_o^{t+1}(x^t, y^t)} \right) \left(\frac{E_o^t(x^{t+1}, y^{t+1})}{E_o^{t+1}(x^t, y^t)} \right) \right]^{\frac{1}{2}}. \quad (8)$$

$E_o^t(t)$ and $E_o^{t+1}(t+1)$ are used in various contexts, with models of relative size to the technology frontier derived from DEA concepts $E_o^t(t+1)$ and $E_o^{t+1}(t)$ derived from the concept of DAE super-efficiency for relative size to the technology frontier.

3. Congestion

The question that arises in a production process is whether the consumption of more resources and inputs increases the results and output. The answer to this question is related to the technology that controls the production activity. In some areas of production, excessive use of resources not only does not improve or change results, but actually reduces them. This condition is called input bottleneck. For example, irrigation in tomato production contributes to some extent to optimal growth, while excessive irrigation destroys growth.

The concept of congestion in DEA is modeled by removing the input feasibility from the production possibility set T_o and forming the set T_{convex} .

The production possibility sets T_o and T_{convex} are as follows:

$$T_o = \left\{ (x, y) \mid x \geq \sum_{j=1}^n \lambda_j x_j, y \leq \sum_{j=1}^n \lambda_j y_j, \sum_{j=1}^n \lambda_j = 1, \lambda_j \geq 0 \right\}. \quad (9)$$

$$T_{convex} = \left\{ (x, y) \mid x = \sum_{j=1}^n \lambda_j x_j, y \leq \sum_{j=1}^n \lambda_j y_j, \sum_{j=1}^n \lambda_j = 1, \lambda_j \geq 0, j = 1, \dots, n \right\}. \quad (10)$$

φ^* is the optimal value of the objective function BCC and γ^* is its optimal value with the difference that the first constraint is the same.

If $\varphi^* \geq \gamma^*$, then DMU has an congestion.

If $\varphi^* = \gamma^*$, then DMU has no congestion.

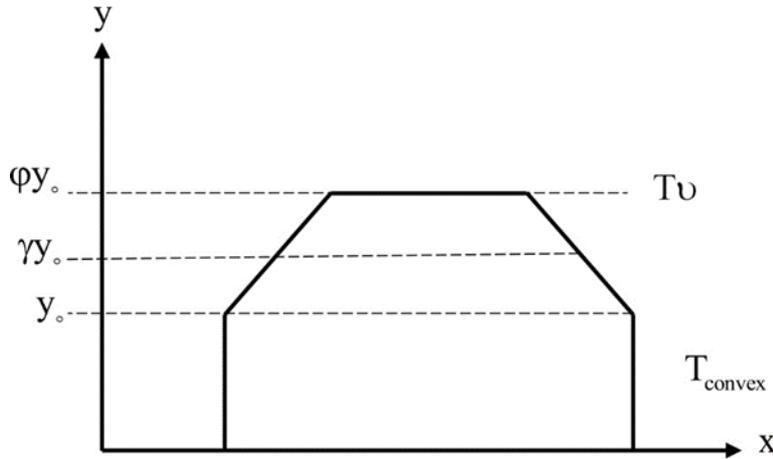


Fig 2 - T_U and T_{convex} frontiers.

4. congestion in progress and regress:

Not only do technology and efficiency change over time, but the optimal use of resources can change as well. Approaching optimal resource use is one component of the progress criterion. Appropriate resource use is permissible to the extent that performance has little or no congestion. The purpose of this section is to present a model that calculates congestion changes by changing the relative distances of units in the Malmquist productivity index. Given the attitude of DEA toward congestion, removing the principle of feasibility of inputs from T_U at a given time leads to two bounds reached by T_U and T_{convex} . The distance between these two limits is measured by the following criteria:

$$D_o^i(j) = (e_o^i(j))^2 / c_o^i(j) \quad (11)$$

$(e_o^i(j)) / c_o^i(j)$ is a measure of traffic congestion. Clearly, the value of this fraction is at least one, and the lower it is, the lower the assessed congestion, which is not present if it is equal to one.

Basically, $e_o^i(j)$ is the relative efficiency of the unit with respect to the technology at time i , which is a criterion for the relative distance to the technology in the Malmquist index. In calculating the progress and regress of units by applying this criterion in the Malmquist index structure, two categories of distances are effectively considered. The model resulting from the application of the above criteria in the Malmquist index is as follows:

$$MI_o = \left(\frac{e^{t+1}(t+1)}{e^t(t)} \right) \left[\frac{\frac{e^{t+1}(t+1)}{c^{t+1}(t+1)}}{\frac{e^t(t)}{c^t(t)}} \right] \left[\frac{e^{t+1}(t)}{e^{t+1}(t+1)} \times \frac{e^t(t)}{e^t(t+1)} \right]^{\frac{1}{2}} \left[\frac{\frac{e^{t+1}(t)}{c^{t+1}(t)}}{\frac{e^{t+1}(t+1)}{c^{t+1}(t+1)}} \times \frac{e^t(t)}{e^t(t+1)}}{\frac{e^{t+1}(t+1)}{c^{t+1}(t+1)}}{\frac{e^t(t)}{c^t(t)}} \right]^{\frac{1}{2}} \quad (12)$$

This criterion can be broken down as follows:

The first component $\frac{e^{t+1}(t+1)}{e^t(t)}$ is the change in efficiency, $\frac{e^{t+1}(t+1)}{c^{t+1}(t+1)}$ is the change in

congestion, the third component $\left[\frac{e^{t+1}(t)}{e^{t+1}(t+1)} \times \frac{e^t(t)}{e^t(t+1)} \right]^{\frac{1}{2}}$ is the change in technology and

the fourth component $\left[\frac{\frac{e^{t+1}(t)}{c^{t+1}(t)}}{\frac{e^{t+1}(t+1)}{c^{t+1}(t+1)}} \times \frac{\frac{e^t(t)}{c^t(t)}}{\frac{e^t(t+1)}{c^t(t+1)}} \right]^{\frac{1}{2}}$ is the congestion of technology within

the period t to t+1.

$MI_0 > 1$ indicates progress, $MI_0 < 1$ indicates regression, and $MI_0 = 1$ indicates no change in productivity.

5. Example:

This example examines 19 listed banks with two inputs and two outputs from 2015 to 2019. The data of these 19 banks were taken from the Codal website, which is shown in Table (1). Following the research based on the banks' financial statements, which include balance sheets and income statements, some results were obtained considering the indices listed below as inputs and outputs.

Input indices	
Input 1	Input 2
Employee costs	Demand deposits, etc.
Other management costs	Savings deposits, etc.
Bad debt expense	Long-term investment deposits
Financial costs	
Cost of commissions	
Total costs	Total deposits

Table 1 Input index

Output indicators	
Output 1	Output 2

Shared revenues	Receivables from the central bank
Mutual income	Receivables from banks and other credit institutions
	Receivables from the government
	Banking finance and receivables from government entities other than banks
	Banking finance and receivables from non-governmental entities other than banks
	in shares and other securities Investments
	in real estate Debts
	to the Central Bank and the National Development Fund Debts
	to banks and other credit institutions
Total revenues	Total assets

Table 2 Output index

		Day	Mellat	Saderat	Tejarat	Pasargad	Parsian	Ayandeh	Eghtesad Novin	Shahr	Melal
2015	input1	11529858	69702029	66680423	43803769	15217649	11264203	42443663	10164946	20523134	3E+06
	input2	1.65E+08	1.15E+09	1.01E+09	7.75E+08	4.17E+08	34004768	462824631	323547281	2.91E+08	8E+07
	output1	41900903	2.43E+08	1.78E+08	1.54E+08	93009666	98694318	111197029	69421048	69392571	2E+07
	output2	1.44E+08	1.83E+09	1.25E+09	8.55E+08	4.59E+08	5.73E+08	562909386	460457022	2.35E+08	6E+07
2016	input1	21068106	70564578	78292558	55501893	26772964	9953123	57818339	8792348	19431312	3E+06
	input2	2.1E+08	1.2E+09	1.09E+09	9.17E+08	4.92E+08	33116735	699108827	344071187	4.31E+08	1E+08
	output1	46498787	2.85E+08	2.04E+08	1.84E+08	87987267	1.01E+08	161129484	62897264	89455304	2E+07
	output2	1.74E+08	2E+09	1.41E+09	2.03E+09	5.99E+08	5.96E+08	665855643	420573908	3.33E+08	1E+08
2017	input1	46431895	94376876	95636697	61279438	35907214	22252814	101544140	11541001	29974828	8E+06
	input2	2.01E+08	1.47E+09	1.26E+09	1.06E+09	6.24E+08	7.51E+08	926142854	328484080	6.67E+08	7E+07
	output1	63169120	3.52E+08	1.94E+08	1.56E+08	1.2E+08	1.33E+08	223842223	59278688	84376780	2E+07
	output2	2.19E+08	2.46E+09	1.93E+09	1.19E+09	6.97E+08	8.46E+08	1196970015	390986028	4.09E+08	2E+08
2018	input1	41633843	1.02E+08	1.12E+08	80741286	41771864	23785170	70182533	13352248	27015045	2E+07
	input2	2.42E+08	2.04E+09	1.58E+09	1.35E+09	8.37E+08	9.83E+08	1302005859	338409041	7.82E+08	1E+08
	output1	48710836	4.37E+08	2.34E+08	2.24E+08	1.54E+08	1.66E+08	168568069	56710914	91996847	3E+07
	output2	2.53E+08	3.87E+09	2.42E+09	1.57E+09	1.75E+09	1.29E+09	1557969210	460737088	4.6E+08	3E+08
2019	input1	26651490	1.86E+08	1.23E+08	89421205	43014099	24715090	30211037	14057538	19025925	2E+07
	input2	2.79E+08	2.58E+09	2.2E+09	1.88E+09	1.15E+09	1.14E+09	1859487090	498108697	8.55E+08	1E+08
	output1	35768661	5.04E+08	2.87E+08	2.65E+08	1.95E+08	1.8E+08	336329066	64352191	99502277	5E+07
	output2	2.51E+08	5.2E+09	2.95E+09	1.97E+09	1.37E+09	8.23E+08	1507145460	606567291	5.95E+08	3E+08

Table 3 Extracted data of banks from Codal website

		Middle East	Post Bank	Saman	Iran Zamin	Karafarin	Sarmayeh	tourism	Resalat
2015	input1	975775	4255733	46082446	7307021	3184755	10863473	4453649	3041400
	input2	32610943	46846211	2.06E+08	108539367	87072471	150529323	1.02E+08	66081419
	output1	7372436	9475105	78269914	22510933	22142306	27714787	24485803	3071294
	output2	37414901	55084631	2.41E+08	40941836	109945832	162124744	47347516	57229192
2016	input1	2299056	4677252	63133043	3630134	3558378	31254930	5801432	4321618
	input2	60673841	51587504	2.24E+08	152109480	105547929	187655708	1.67E+08	94798527
	output1	10225138	9667231	93249682	29425536	20129444	19509596	33909435	4624145
	output2	69469443	62692625	1.99E+08	45817943	127167814	175092566	81443423	80059545
2017	input1	2910760	7370048	99175474	6585818	4799638	33272141	3038983	4876168
	input2	75835118	61352846	2.74E+08	172943105	132594714	184554128	2.49E+08	1.27E+08
	output1	12589880	13433271	1.33E+08	38557330	22246835	16938721	12007112	5083473
	output2	91821830	105562504	2.3E+08	94391017	137544898	221361921	1.66E+08	1.05E+08
2018	input1	3470470	9771392	93650364	17974312	4828276	51071669	5341180	7103887
	input2	109961870	87768050	3.56E+08	228699658	276713323	172869340	3.5E+08	1.5E+08
	output1	17615194	15228612	1.36E+08	54766023	24604889	6119285	64167043	6790905
	output2	133103088	178808337	4.7E+08	128071842	151049572	273134687	2.09E+08	1.34E+08
2019	input1	4014360	11040078	14105178	28050269	6181630	53591477	29371497	9160264
	input2	123486758	131342627	4.58E+08	331542314	186681841	178741864	3.84E+08	2.3E+08
	output1	23460301	21680392	1.97E+08	25379580	28814220	7202312	93625784	10044766
	output2	250620728	211040018	4.67E+08	126947809	194003781	319989292	3.71E+08	1.8E+08

Table 4 Extracted data of banks from Codal website

It should be noted that the financial statements of Shahr, Iran Zamin and Melal banks have not been reviewed on the Codal website.

DMUs	2015			
	e BCC	e convex	e BCC/e convex	Congestion
Day	2.37103	2.15046	1.102568753	Positive
Mellat	1	1	1	Negative
Saderat	1.26416	1	1.26416	Positive
Tejarat	1.15804	1.11858	1.035276869	Positive
Pasargad	1.16576	1	1.16576	Positive
Parsian	1	1	1	Negative
Ayandeh	1.38613	1.2574	1.2574	Positive
Eghtesad Novin	1.12085	1	1.12085	Positive
Shahr	1.75074	1.6928	1.034227316	Positive
Melal	1.20227	1.04193	1.04193	Positive
Sina	1.22969	1	1.22969	Positive
Middle East	1	1	1	Negative
Post Bank	3.78104	3.73295	3.73295	Positive
Saman	1.54476	1	1.54476	Positive
Iran Zamin	2.82395	2.61498	2.61498	Positive
Karafarin	1.21847	1.0635	1.145716972	Positive
Sarmayeh	3.40782	3.14518	1.083505555	Positive

Tourism	1.56183	1.38584	1.126991572	Positive
Resalat	2.53401	2.4226	2.4226	Positive

Table 5 Calculated results to determine the congestion status in 2015

DMUs	2016			
	e BCC	e convex	e BCC/e convex	congestion
Day	2.7711	2.73483	1.01326225	Positive
Mellat	1	1	1	Negative
Saderat	1.30965	1	1.30965	Positive
Tejarat	1	1	1	Negative
Pasargad	1.68641	1.55559	1.08409671	Positive
Parsian	1	1	1	Negative
Ayandeh	1.27951	1.20764	1.05951277	Positive
Eghtesad Novin	1.22763	1	1.22763	Positive
Shahr	1.45007	1.20041	1.20797894	Positive
Melal	1	1	1	Negative
Sina	1.53141	1.38766	1.10359166	Positive
Middle East	1	1	1	Negative
Post Bank	1	1	1	Negative
Saman	1.40689	1	1.40689	Positive
Iran Zamin	1	1	1	Negative
Karafarin	1.22775	1.10598	1.11010145	Positive
Sarmayeh	4.84048	3.59518	1.34638043	Positive
Tourism	1.59218	1.45072	1.0975102	Positive
Resalat	2.60623	2.44483	1.06601686	Positive

Table 6 Calculated results for determining congestion status in 2016.

DMUs	2017			
	e BCC	e convex	e BCC/e convex	congestion
Day	1.22734	1.22734	1	Negative
Mellat	1	1	1	Negative
Saderat	1.10032	1.06222	1.035868276	Positive
Tejarat	1.40279	1.40279	1	Negative
Pasargad	1.20525	1.20525	1	Negative
Parsian	1	1	1	Negative
Ayandeh	1.12485	1	1.12485	Positive
Eghtesad Novin	1.02988	1.02988	1	Negative
Shahr	1.63975	1.63975	1	Negative
Melal	1	1	1	Negative
Sina	1.04054	1.04054	1	Negative
Middle East	1	1	1	Negative
Post Bank	1	1	1	Negative
Saman	1	1	1	Negative
Iran Zamin	1	1	1	Negative
Karafarin	1.07846	1.07846	1	Negative
Sarmayeh	1.74822	1.42533	1.226537012	Positive
Tourism	1	1	1	Negative
Resalat	1.53782	1.53782	1	Negative

Table 7 Calculated results for determining the congestion status in 2017

DMUs	2018			
	e BCC	e convex	e BCC/e convex	congestion
Day	1.60114	1.52175	1.05217	Positive
Mellat	1	1	1	Negative
Saderat	1.43358	1.41219	1.01514669	Positive
Tejarat	1.32632	1.32189	1.00335126	Positive
Pasargad	1.35511	1.35122	0.74220251	Positive
Parsian	1	1	1	Negative
Ayandeh	1.29222	1.28974	1.00192287	Positive

Eghtesad Novin	2.35963	1	2.35963	Positive
Shahr	2.0813	2.07011	1.00540551	Positive
Melal	1	1	1	Negative
Sina	1.60957	1.60957	1	Negative
Middle East	1	1	1	Negative
Post Bank	1.35176	1.35176	1	Negative
Saman	1	1	1	Negative
Iran Zamin	2.31754	2.31556	1.00085508	Positive
Karafarin	1.61783	1.53374	1.05482676	Positive
Sarmayeh	1.80524	1	1.80524	Positive
Tourism	1.6729	1	1.6729	Positive
Resalat	3.06712	3.06712	1	Negative

Table 8 Calculated results to determine the congestion status in 2018

DMUs	2019			
	e BCC	e convex	e BCC/e convex	congestion
Day	2.1542	2.1542	1	Negative
Mellat	1	1	1	Negative
Saderat	1.24141	1.24141	1	Negative
Tejarat	1.41675	1.41675	1	Negative
Pasargad	1.10457	1.10457	1	Negative
Parsian	1.30178	1.30178	1	Negative
Ayandeh	1	1	1	Negative
Eghtesad Novin	1.01879	1.01879	1	Negative
Shahr	1.45811	1.45811	1	Negative
Melal	1	1	1	Negative
Sina	1.64012	1.64012	1	Negative
Middle East	1	1	1	Negative
Post Bank	1.27669	1	1.27669	Positive
Saman	1	1	1	Negative
Iran Zamin	4.25366	4.10114	1.03718	Positive
Karafarin	1.59662	1.59662	1	Negative
Sarmayeh	1.15786	1	1.15786	Positive
Tourism	1.47085	1.3855	1.0616	Positive
Resalat	2.25495	2.25495	1	Negative

Table 9 Calculated results for determining the congestion status in 2019

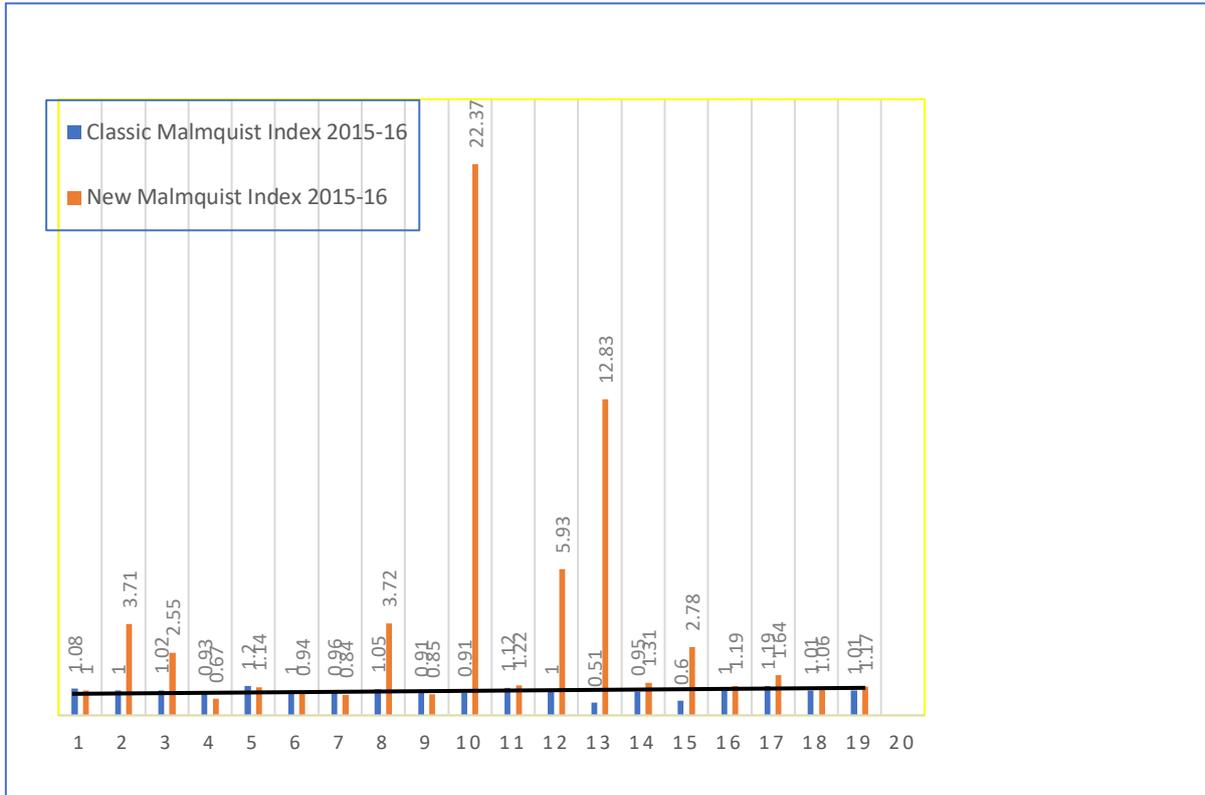


Fig. 3 Assessment of progress and regression in bank productivity in 2015-16.

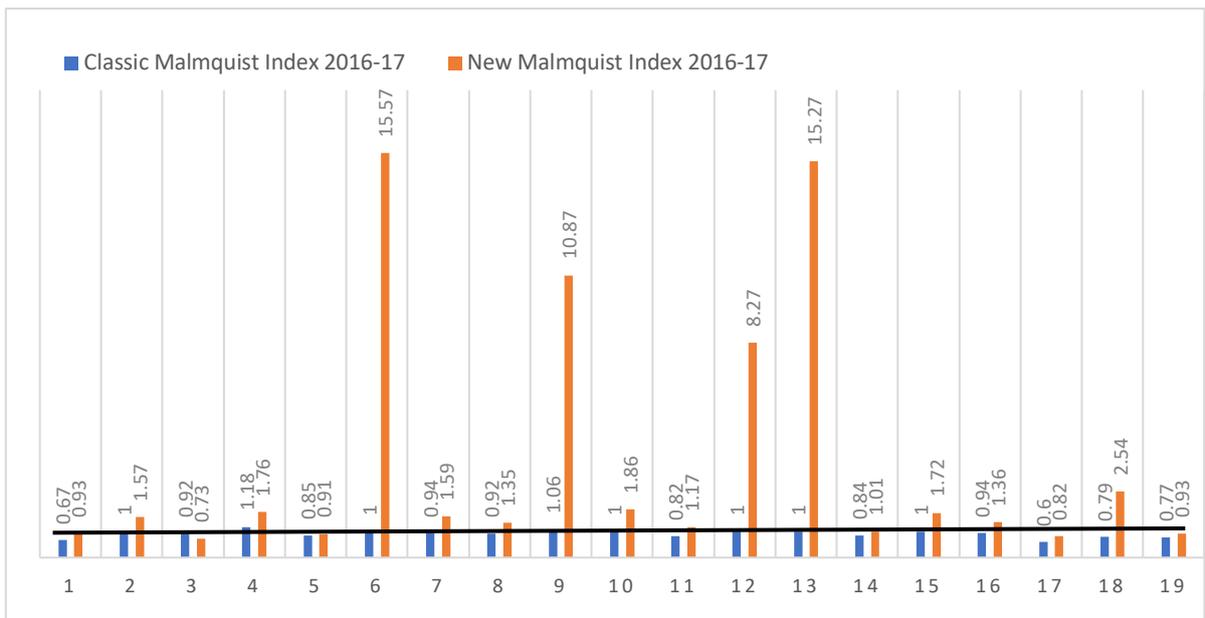


Fig. 4 Assessment of progress and regression in bank productivity in 2016-17

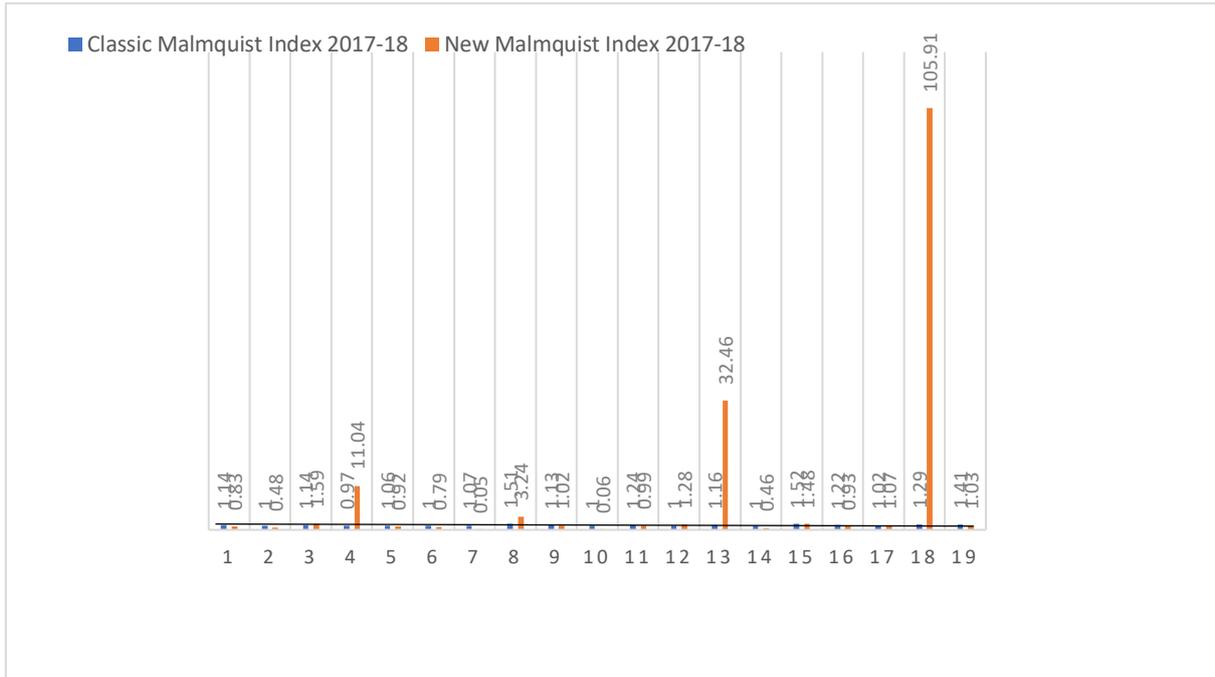


Fig. 5 Assessment of progress and regression in bank productivity in 2017-18

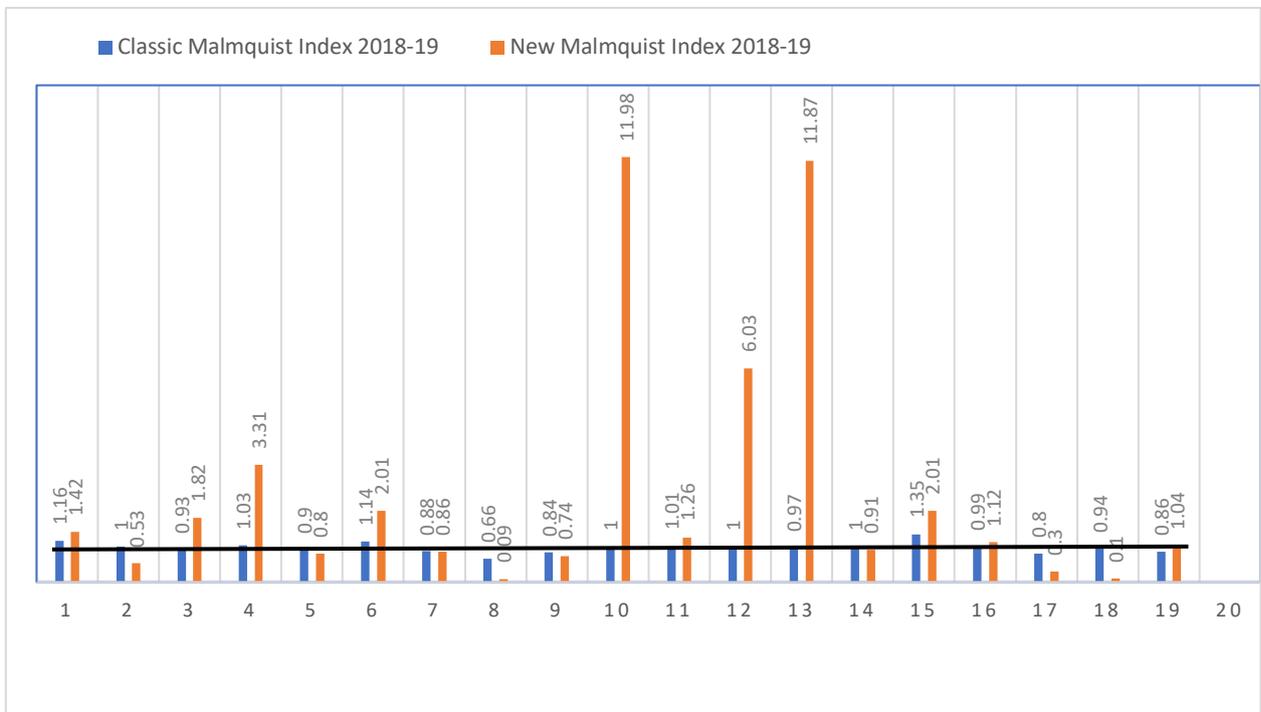


Fig. 6 Assessment of progress and regression in bank productivity in 2018-19

In 2017-18, Ayandeh Bank shows a regression according to the traditional Malmquist index, as the value of the Technological Change component is 0.93, lower than the other components. However, in the new Malmquist index, the values of congestion and technological change are 0.89 and 0.05, respectively, and these two components also have lower values than the other components. At the same time, the new index shows progress. Therefore, the new Malmquist index is able to identify the components of progress.

In 2017-18, according to the traditional Malmquist index, Tejarat Bank shows progress because the value of the technological change component is 1.03, which is higher than the other components. However, in the new index, the values of congestion change and technological change are 1 and 12.3, respectively, which is high. Therefore, the new Malmquist index is declining.

Similarly, Saderat Bank 2018-19 shows progress according to the traditional Malmquist index, as the value of the technological change component is high at 0.07. However, the values of congestion and technological change in the new index are 0.99 and 2.05, respectively, which are high. Therefore, the new Malmquist index is declining. The reason for this difference is that the traditional Malmquist index measures only efficiency and technology changes over time, while the new Malmquist index can measure congestion and congestion technology changes in addition to efficiency and technology changes.

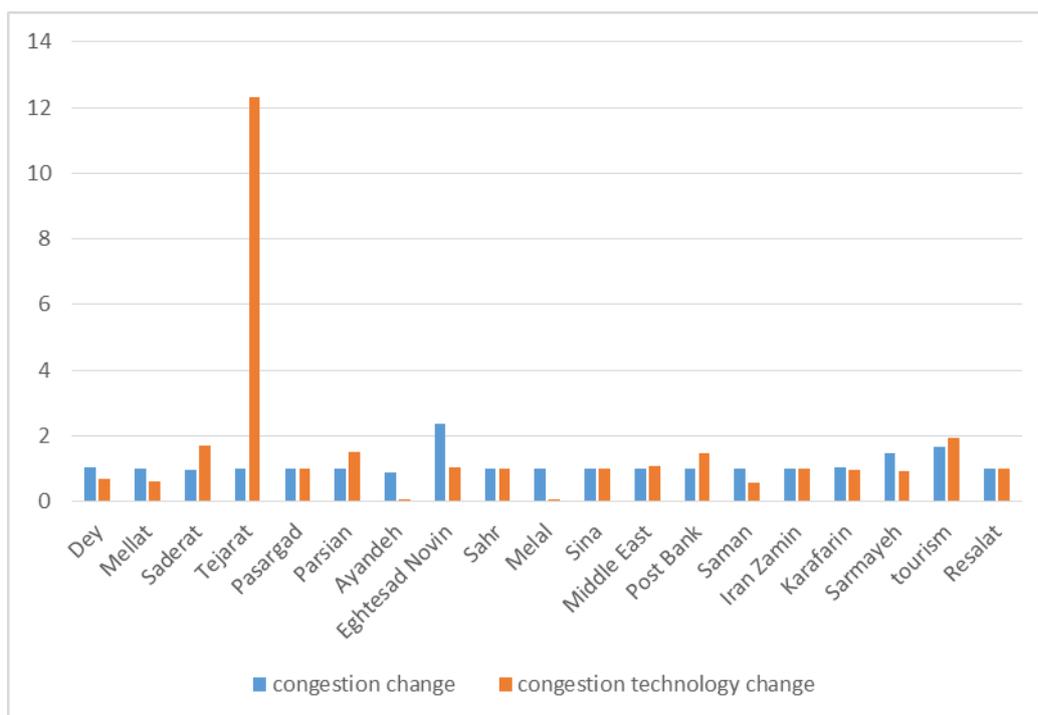


Fig.7 Evaluation of changes in congestion and congestion technology in the new index

6. Conclusion

This paper presents the new Malmquist productivity index, which is able to measure the progress and regress of decision-making units over time based on the three criteria of efficiency change, technology change, and congestion change, while only efficiency change and technology change were considered in the traditional Malmquist index, and the congestion of production technology was not considered in the traditional Malmquist index. The extracted data are the banks listed on Tehran Stock Exchange from Kodal website, which clearly show the difference between the traditional Malmquist index and the new Malmquist index.

For example, Future Bank with the traditional Malmquist index based only on efficiency in two periods has a decline, while the new Malmquist index based on efficiency and congestion in two periods has advanced with the decline in density changes and also the changes in

congestion technology. shows.
Therefore, by applying the change in congestion and the change in congestion technology, the new index provides the opportunity to evaluate the progress and regression with new components.

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