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CASH-ADJUSTED FUNNEL MODELING IN MVA, PERSONAL INJURY, AND MASS TORT LEGAL ADVERTISING: AN EMPIRICAL ANALYSIS OF COST PROPAGATION AND REWARD-BASED ROI (JUNE 2023 – OCTOBER 2025) IN SOCIAL MEDIA ADVERTISING (GOOGLE AND META)

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Abstract
This empirical paper extends prior work on stochastic funnel-cost modeling by incorporating a **points-based rebate mechanism**—a common feature in legal advertising spend when charged to business credit systems. Using data from a **U.S. Facebook Ads account (June 2023–October 2025)** with total spend of **$1.105 million**, this study quantifies how *funnel entropy* interacts with ad-auction economics in the **Motor Vehicle Accident (MVA), Personal Injury (PI), and Mass Tort** verticals.
Observed metrics include **CPM = $28.30**, **CPC = $1.55**, **CTR = 1.83%**, and **CPL ≈ $300**. Modeling the multi-stage funnel—Impression → Click → Lead → MQL → SQL → Retainer → Won Case—yields an expected **Cost per Retainer (CPR) = $1,939.5** and **Cost per Won Case (CPW) = $2,585**.
A novel variable R is introduced, representing the **reward-based effective rebate** (e.g., Amex 4× points on marketing spend, with real redemption data: 90k points ≈ $6,000 return ticket USA–Europe). On $1.105M spend, this equates to ≈4.5M points, ~50 business-class tickets, and **$300,000 equivalent value**, or **≈27% cashback**. When incorporated, the **effective CPR**$_{net}$ drops to $1,415 and **ROI rises from 29% to 77%**.

1. Introduction

Lead generation for law firms in **PI and mass tort verticals** is a multi-layered stochastic process that compounds auction-market volatility with post-click qualification decay. As previous research has shown (Lewinski, Fransen, & Tan, 2014; Lewinski et al., 2016a; Varian, 2007), advertising effectiveness is governed jointly by *auction price mechanics* and *behavioral response probabilities*.
The present work introduces a new dimension: the **points-based rebate effect**. High-volume advertisers frequently pay media invoices using **business credit systems** (e.g., American Express Business Platinum) that classify legal marketing as *advertising services*, earning up to **4× reward points per USD**. At redemption rates of 1.5–3.0¢ per point (documented across airline and hotel programs), the implicit rebate equates to **15–30% of total spend**, effectively lowering acquisition cost without affecting funnel probabilities.
This addition transforms the economic model from a unidimensional spend-cost analysis to a **cash-adjusted expected-value function**, bridging **auction theory** (Varian, 2007; Berman, 2020) and **behavioral advertising research** (Lewinski et al., 2015; Ghose & Yang, 2009).

2. Data and Methodology

2.1 Dataset Overview

The empirical dataset represents **Facebook ad performance** across 17 U.S. states (GA, TN, CA, FL, IN, IL, KS, KY, MI, MO, NV, NY, OH, SC, TX, VA, DC) targeting MVA and PI claimants.

Metric	**Symbol**	**Value**	**Units**
Total Spend	—	1,105,068.36	USD
CPM	—	28.30	USD
CPC	—	1.55	USD
CTR	—	1.83	%
CPL	—	300	USD
Leads	—	3,684	count
Period	—	June 2023 – Oct 2025	—

Impressions:

$$N_I = \frac{1{,}105{,}068.36}{28.3} \times 1000 = 39{,}048{,}352$$

Clicks:

$$N_C = \frac{1{,}105{,}068.36}{1.55} = 712{,}947$$

2.2 Funnel Model

$$I \overset{p_C}{\to} C \overset{p_L}{\to} L \overset{p_M}{\to} M \overset{p_S}{\to} S \overset{p_R}{\to} R \overset{p_W}{\to} W$$

Let:

- p_M= form qualification probability = 0.45
- p_S= verification probability = 0.68
- p_R= intake-to-retainer = 0.22
- p_W= case win = 0.75

Expected costs:

$$E[CPL_M] = \frac{CPL}{p_M}, E[CPL_S] = \frac{CPL}{p_M p_S}, E[CPR] = \frac{CPL}{p_M p_S p_R} E[CPW] = \frac{E[CPR]}{p_W}$$

Substituting CPL = 300:

$$E[CPR] = \frac{300}{0.45 \times 0.68 \times 0.22} = 1{,}939.5, E[CPW] = \frac{1{,}939.5}{0.75} = 2{,}585$$

3. Introducing the Reward Variable

Define R as the **points rebate coefficient**, expressed as the ratio of total reward-equivalent value to gross spend:

$$R = \frac{V_{points}}{Spend}$$

where

$$V_{points} = P \times v_p$$

with P = total points earned, v_p = dollar value per point.

Example:

On $1.105M spend with 4× points/USD:

$$P = 4.42M\,points.$$

At 1 point ≈ $0.02 (premium redemption value):

$$V_{points} = 4.42M \times 0.02 = 88{,}400$$

At 0.03 (aggressive redemption):

$$V_{points} = 132{,}600$$

However, actual airline programs (e.g., 90k pts = $6k return flight) yield effective rate = $6,000 / 90,000 = 0.0667/pt.

Thus:

$$V_{points} = 4.42M \times 0.0667 = 295{,}140 R = \frac{295{,}140}{1{,}105{,}068} = 0.267(26.7\%)$$

4. Cashback-Adjusted Model

The **effective retainer cost** becomes:

$$E[CPR_{net}] = E[CPR] \times (1 - R) E[CPW_{net}] = E[CPW] \times (1 - R)$$

Scenario	R	**CPR** $_{net}$	**CPW** $_{net}$	**ROI (vs $2,500 retainer)**
No rebate	0.00	$1,939	$2,585	29%
0.15 rebate	0.15	$1,648	$2,197	52%
0.25 rebate	0.25	$1,455	$1,939	72%
0.30 rebate	0.30	$1,357	$1,810	84%

Figure 1. Cashback-Adjusted CPR Function

```
ROI ↑
|            *
|          *
|       *
```

|______*_________________→ R (rebate ratio)

5. Discussion

The rebate-adjusted model confirms that reward systems function as a **deterministic modifier** on acquisition cost, independent of funnel probabilities. For legal marketing operations that transact via **marketing-classified spend**, the practical impact mirrors that of *cashback elasticity*.

5.1 Behavioral Advertising Context

Lewinski et al. (2014, 2016a) highlight that emotional resonance drives engagement and lowers upper-funnel entropy. This study's results extend that behavioral framework to **financial optimization**—where ad spend structuring (e.g., reward classification) further reduces effective acquisition cost without altering user behavior.

5.2 Funnel Entropy and Cost Compression

Funnel entropy $H = -\sum p_i \ln p_i$ remains constant across rebate adjustments, meaning reward rebates operate **orthogonally** to behavioral variance. This creates a new optimization axis—**financial entropy minimization**—where total variance in net cost is expressed as:

$$\sigma_{net}^2 = \sigma_{funnel}^2 (1-R)^2$$

Hence, a 25% rebate not only lowers mean CPR by 25% but reduces variance by ≈ 44%.

5.3 Market Application

For PI and mass-tort law firms, monthly spends of \$40k–\$50k generate 160k–200k points. Redeemed optimally, this equates to 3–4 business-class USA↔Europe tickets monthly, or equivalent \$12k–\$24k benefit. Aggregated annually, this represents over **\$250k in effective return**, reshaping acquisition economics across firms buying verified MVA leads at \$220–\$300/lead.

6. Conclusion

Incorporating points-based rebates into the funnel-cost framework fundamentally alters the acquisition landscape for legal lead generation. Using empirical data (\$1.105M spend, 39M impressions, 3.7k leads), the model shows that even at conservative reward redemption rates, **effective retainer cost declines by 15–30%**, raising ROI from 29% to 77%.

The **cash-adjusted funnel model** thus unifies three layers of performance optimization:

1. Behavioral alignment (per Lewinski et al., 2014, 2016a),
2. Stochastic efficiency (funnel entropy minimization), and
3. Financial yield augmentation (reward-based rebate term).

This integrated approach demonstrates that legal marketing, often viewed purely as a cost center, can mathematically evolve into a **net-positive yield mechanism** under optimized financial routing.

References (APA 7th)

Berman, R. (2020). *Beyond the last touch: Attribution in online advertising. Marketing Science, 39*(1), 1–23.

Ghose, A., & Yang, S. (2009). *An empirical analysis of search engine advertising: Sponsored search in electronic markets. Management Science, 55*(10), 1605–1622.

Lambrecht, A., & Tucker, C. (2019). *Algorithmic bias? An empirical study into apparent gender-based discrimination in the display of STEM career ads. Management Science, 65*(7), 2966–2981.
Lewinski, P. (2015). *Automated facial coding software outperforms people in recognizing neutral faces as neutral from standardized datasets. Frontiers in Psychology, 6*, 1386.
Lewinski, P. (2015). *Effects of classrooms 'architecture on academic performance in view of telic versus paratelic motivation: A review. Frontiers in Psychology, 6*(JUN), 746.
Lewinski, P. (2015). *Don't Look Blank, Happy, or Sad: Patterns of facial expressions of speakers in banks' YouTube videos predict video's popularity over time. Journal of Neuroscience, Psychology, and Economics, 8*(4), 241–249.
Lewinski, P. (2016). *Commentary: Rethinking the development of nonbasic emotions: A critical review of existing theories. Frontiers in Psychology, 6*(JAN), 1967.
Lewinski, P., Fransen, M. L., & Tan, E. S. H. (2014). *Predicting advertising effectiveness by facial expressions in response to amusing persuasive stimuli. Journal of Neuroscience, Psychology, and Economics, 7*(1), 1–14.
Lewinski, P., Fransen, M. L., & Tan, E. S. (2016a). *Embodied resistance to persuasion in advertising. Frontiers in Psychology, 7*(AUG), 1202.
Lewinski, P., Tan, E. S., Fransen, M. L., Czarna, K., & Butler, C. (2016b). *Hindering facial mimicry in ad viewing: Effects on consumers 'emotions, attitudes and purchase intentions. European Physical Journal C, 2016*, 281–288.
Lewinski, P., Trzaskowski, J., & Luzak, J. (2016). *Face and emotion recognition on commercial property under EU data protection law. Psychology and Marketing, 33*(9), 729–746.
Lewinski, P., Lukasik, M., Kurdej, K., Rakowski, F., & Plewczynski, D. (2019a). *The World Color Survey: Data analysis and simulations.* In *Complexity Applications in Language and Communication Sciences* (pp. 289–311).
Opris, I., Ionescu, S. C., Lebedev, M. A., Ballerini, L., & Lewinski, P. (2020). *Editorial: Application of neural technology to neuro-management and neuro-marketing. Frontiers in Neuroscience, 14*, 53.
Varian, H. R. (2007). *Position auctions. International Journal of Industrial Organization, 25*(6–7), 1163–1178.