# FRAYER MODEL TO DEVELOP GEOMETRIC CONCEPTS AMONG NINTH GRADE STUDENTS IN THE GALILEE REGION

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**Abstract:** The primary objective of this research is to examine the influence of the Frayer Model in enhancing geometric concepts among Arab minority's ninth-grade students from the Galilee region in Israel. The research encompassed 50 ninth-grade students from a middle school in the Galilee, divided into two groups: a control group of 24 students, taught the unit "Angles and Arcs in the Circle" through traditional methods, and an experimental group of 26 students, exposed to the same unit using the Frayer Model. Both groups were matched in age and previous academic achievement. The researcher devised an achievement test in geometric concepts, aligned with Bloom's classification of objectives in the areas of remembering, understanding, and application. The test's validity and stability were ensured before administering it to both research groups before and after the study. The post-application results revealed a statistically significant difference in overall achievement at the three Bloom levels, favoring the experimental group taught with the Frayer Model.

**Keywords:** Frayer Mode, Geometric Concepts, Angles in a Circle, Mathematics Education

## Introduction

Mathematics plays a fundamental and indispensable role in human existence, serving as a crucial tool for quantification, comparison, and comprehension of the world around us. Despite its pervasive presence in various aspects of daily life, its significance often goes unnoticed. Mathematics extends beyond mere calculations, permeating diverse fields such as art, music, science, and technology, thereby shaping communication, problem-solving, and decision-making processes. Throughout history, mathematics has remained integral to human progress, contributing profoundly across multiple disciplines.

Among the numerous fields that crucially depend on mathematics, engineering has garnered substantial scholarly attention, particularly in educational literature. Koehler et al. (2014) highlight its importance due to the distinct teaching and learning requirements at various educational stages. Engineering education not only imparts practical life skills but also integrates multiple mathematical principles, establishing meaningful connections between theoretical knowledge and real-world applications (Vinnervik, 2022; Hibi, 2017; Hibi, 2018). Within engineering, geometry holds a central role, providing foundational principles essential for geometric construction. The development of geometric concepts is therefore a crucial objective in geometry education, as it fosters the acquisition of critical engineering skills (Wagner et al., 2020). Given that mathematical concepts are inherently multifaceted; the

process of concept formation plays a vital role in cognitive development. A well-structured understanding of these concepts enables learners to distinguish between key principles and related notions, facilitating both generalization and differentiation. This, in turn, enhances the efficiency of the learning process by conserving cognitive resources (Assadi, N., & Hibi, W, 2020; Renzulli, 2023).

Recognizing the limitations of conventional instructional methodologies, particularly in mathematics, educators have increasingly explored more effective pedagogical approaches. Active learning has emerged as a promising alternative, driven by the challenges students face in assimilating new information within traditional teaching settings. Learners often experience confusion and uncertainty following passive instruction, as they struggle to integrate newly acquired knowledge into their cognitive frameworks. This issue, as noted by Christie and De Graaff (2017), underscores the need for innovative instructional strategies that actively engage students in the learning process.

The Frayer Model is an active learning framework developed by Dorothy Frayer, Klausmeier, and Fredrich at the University of Wisconsin in 1969. It aims to enhance concept learning based on Bruner's ideas. The model has five key objectives, with a primary focus on concept mastery. It assists students in concentrating on a concept by exploring its defining traits and examples. Information is structured in a table format, featuring a central circle for the concept, surrounded by a rectangle divided into four squares. Students define the concept, list its characteristics, and provide both relevant and non-relevant examples. This visual approach supports students in gaining a comprehensive understanding of the concept (Hibi 2024c; Smith-Walters, Bass, & Mangione, 2016).

While guiding mathematics instruction in the Galilee region, the study found that middle and high school students' comprehension of geometric concepts was noticeably lacking. The current study fills a specific need by methodically examining the efficacy of the Frayer Model as an instructional intervention, even though teacher observations have anecdotally recognized this issue. In addition to advancing theoretical knowledge of geometric conceptual learning processes, this study provides useful recommendations for enhancing teaching methods that go beyond anecdotal evidence. After comprehensive evaluations of the students' in-class performance, engagement in classroom activities, analysis of examination outcomes, and extensive discussions with fellow educators and mentors. These discussions focused on diverse pedagogical methods—both traditional and contemporary—designed to enhance students' understanding of geometry concepts. This study incorporates the Frayer Model with well-known learning theories like Vygotsky's social constructivist theory, Piaget's stages of cognitive development, and the SOLO taxonomy. These frameworks offer insightful viewpoints on how students organize and create knowledge. By combining these models, a deeper theoretical basis is made possible, enabling a thorough comprehension of how specific teaching techniques might improve conceptual learning in geometry (Assadi, & Hibi, 2022; Bader, 2020).

## Research Problem

The primary research problem revolves around the limited comprehension of geometric concepts among secondary school students. Consequently, there is a need to introduce a novel methodology that facilitates the learning of geometric concepts, ultimately resulting in

improved academic achievement. The specific focus of the research problem can be encapsulated in the following question:

"What is the effect of using the Frayer Model on the development of geometric concepts among Arab ninth grade students in the Galilee region."

## Research Limitations

The research was confined to:

Ninth-grade students at the same school.

One unit of the geometry curriculum is "Angles and Arcs in a Circle."

Measuring the first three levels of Bloom's taxonomy (remembering-understanding-application).

## Research Objective

The aim of this research is to develop geometric concepts among ninth-grade Arab students in the Galilee region using the Frayer Model.

## Significance of the Research

The current research can benefit the following individuals:

Students: Secondary school students can greatly benefit from this research as it aims to enhance their understanding of geometric concepts by providing them with worksheets and activity booklets specifically designed for practicing and reinforcing these concepts.

Teachers: Teachers can benefit from this research by providing them with a teaching model and a teacher's guide that they can use to enhance students' understanding of geometric concepts.

Curriculum developers: It highlights the importance of reorganizing the content to make the most of this teaching model.

Researchers: This research can benefit researchers in identifying new research areas.

## Research Terminology

McNeil (2015) introduces the Frayer Model, a robust teaching approach that consists of three key components: an analytical methodology for deconstructing concepts into their fundamental elements, an effective instructional strategy to facilitate students' understanding, and an assessment tool to evaluate students' mastery and assimilation of the concept. Essentially, this model serves as a comprehensive framework for conceptual learning and acquisition. The Frayer Model has many drawbacks, despite its effectiveness in assisting students in developing conceptual understanding through the organization of definitions, traits, and instances. For example, without further instructional support, it might not adequately address students' misconceptions. Furthermore, unless combined with other teaching methodologies, the model's primary concentration on vocabulary growth may not adequately engage higher-order thinking skills. By recognizing these limitations, the study's scope is made clear and potential areas for further investigation are highlighted.

As asserted by Yao and Manouchehri (2019), geometric concepts encompass mental constructs or abstract mental representations that students develop through the process of generalizing observed characteristics and features from analogous examples of the concept. These concepts may also be delineated as groupings of objects, symbols, topics, elements, or specific

occurrences based on shared attributes or features determined by particular criteria. Geometric concepts manifest as abstract structures derived from experiences or sequential study materials.

Of all the mathematical subjects taught in junior high schools, geometry is thought to be the most complex (Hibi, 2021a; Hibi, 2021b; Gosztonyi, 2022). According to the International Organization of Teachers of Mathematics (2000), its deductive structure and abundance of concepts distinguish it from other mathematical fields and make teaching and studying it especially difficult. This intricacy results from pupils' inadequate geometric reasoning skills, which are crucial for subject mastery.

Tanguay and Venant (2016) studied how children aged 11 to 12 processed angles and how they understood angle measurement. Their analysis showed a hazy and primarily measurement-based link that lacked a strong theoretical basis and mainly relied on iconic representations. "Disintegration" and "amalgamation," two dialectical genetic processes that the researchers discovered, shed light on the connections between different meanings in angle creation.

Angle measures were first interpreted as geometric objects in Moore's (2013) study, which looked at a teaching experiment with pre-calculus pupils. However, by creating an arc length representation of angle measures, students gained a more thorough comprehension of degrees and radians as the study went on. In order to promote a cohesive understanding of angle measurement, it was helpful to quantify angle measure using procedures that involved measuring arc lengths.

**Theoretical Framework**

Because geometric concepts are abstract and require sophisticated spatial thinking, students frequently struggle to understand them. The Frayer Model, which systematically aids conceptual understanding by grouping key features and examples of concepts, is suggested as an instructional solution in this study to solve these issues.

The Input-Process-Output (IPO) model is used to frame this intervention. The Frayer Model is the instructional process, learner challenges are the inputs, and the intended learning outcomes are the outputs in this framework. This technique offers a transparent process for demonstrating how focused instruction might change students' conceptual understanding.

As it clearly engages students in defining, analyzing, and differentiating concepts—all of which are essential for profound understanding—the Frayer Model is thought to be more effective than other teaching methods in geometry. The Frayer Model encourages active cognitive engagement and metacognitive reflection, in contrast to methods that just concentrate on rote memory or procedural tasks.

Drawing from personal experience with high school geometry students, it is observed that many tend to define geometric concepts in a simplistic and unsophisticated manner, often disregarding formal verbal definitions. For instance, students may define a square merely as a quadrilateral with equal straight sides and four right angles, neglecting the more precise definition: a parallelogram with one right angle and a pair of adjacent equal sides (Figure 1).
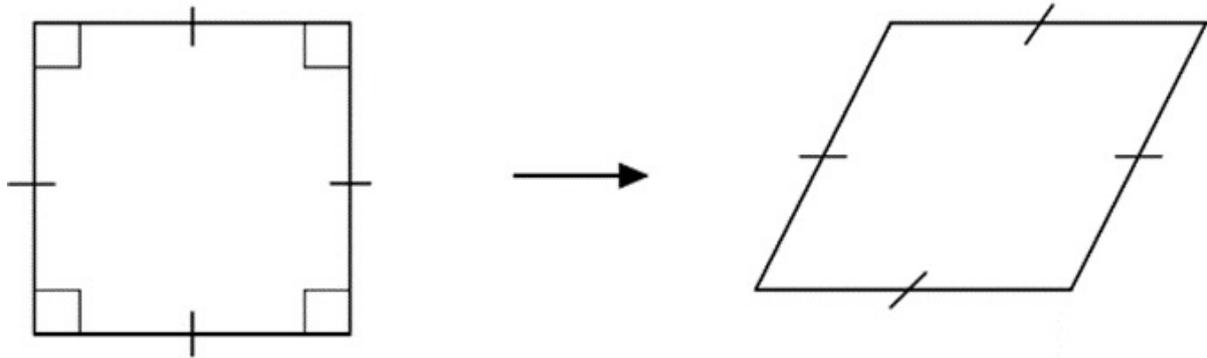
**Figure 1. The square of the parallelogram family**
Source: Surname Year (Godard, 2012)

Some students do not distinguish between the circumference of a semicircle (Figure 2a) and the length of the semicircle arc (Figure 2b). Although many of them understand that the circumference of a circle is the length of a circular band- if the circle is cut and the length of this strip is measured on a ruler-, they do not realize that the diameter of the circle will be part of the circumference of the semicircle. (See Figure 2).
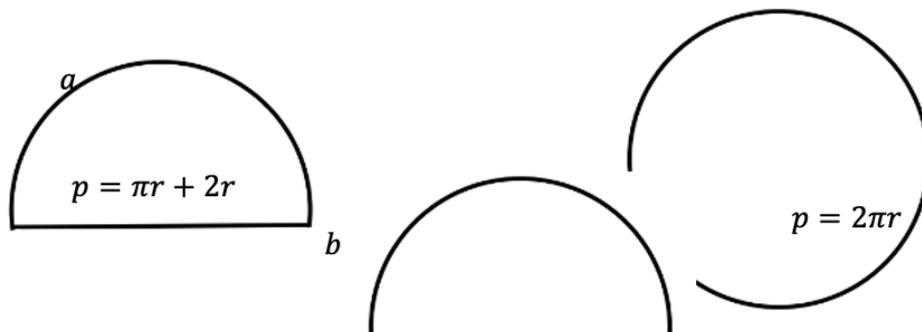


Figure 2. The circumference of a semicircle

Many students miss define the string in a circle, so sometimes when they see any two-line segments that are equal in a circle (without realizing that these pieces are not strings), they mistakenly conclude that the circumferential angles on these two pieces are also equal (see Figure 3).
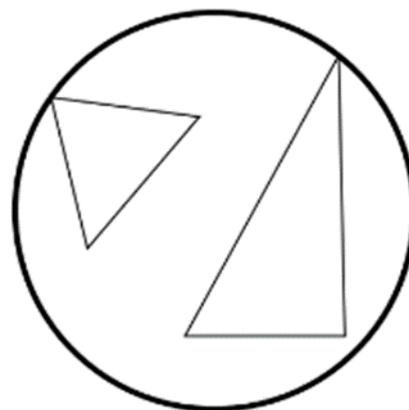
Figure 3. Strings and lines ∝≠β

Many students also make the mistake of using the following correct concept, which asserts, "Two circumferential angles are equal if and only if they rest on the same string and on the same side" and think that the two angles are equal if they rest on the chord at different directions. (See Figure 4).
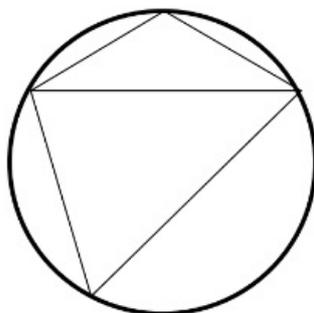
∝≠β



Figure 4. Circumferential angles on the same side

The complexity of geometry, relative to other mathematical fields taught in junior high schools, has been recognized in educational research (Hibi, 2022a; Hibi, 2022b; Gosztonyi, 2022). Its deductive structure and multitude of concepts distinguish it from other domains, presenting unique challenges in teaching and learning. The International Organization of Teachers of Mathematics (2000) highlights the difficulties stemming from students' limited development of geometric thinking, crucial for engaging with the subject effectively.

Students often confuse concepts related to quadrilaterals inscribed by or inscribing a circle, as well as triangles intercepted by or intercepting a circle, due to the distinct characteristics associated with each concept. Addressing such misconceptions is essential to fostering a deep understanding of geometric principles (Hibi, 2021c; Hibi, 2024b; Moore 2013; Tanguay and Venant 2016).

To improve geometry education, innovative teaching methods are crucial, considering the complexity and comprehensive nature of the subject. Thompson and Senk (2014) emphasize the importance of building geometric concepts in a manner that aids students in differentiating between various concepts and achieving a profound level of comprehension.

The teacher's role in internalizing geometric concepts is significant, and the representations they present to students play a vital role in this process. Integrating diverse geometric representations can enhance students' understanding and visualization of concepts, particularly when dealing with angles and their measurements (Alyami 2022; Hibi, 2024a; Hibi, 2024d).

Active learning has emerged as a valuable approach to address the shortcomings of traditional teaching methods, especially in geometry. Jackiw and Sinclair (2009) suggest that students' confusion and lack of integration of new information may stem from conventional teaching practices, which active learning endeavors to remedy.

Gal and Linchevski (2010) emphasize the centrality of geometric concepts in the geometry learning process, as mastering these concepts fosters the development of various skills in geometry.

Numerous studies have identified weaknesses in students' geometric concepts and proposed innovative strategies, such as the paper folding model (Origami), Van Hiele model, and Hofer's geometry model, among others (Komatsu 2017; Sinclair and Yurita 2008).

Technological advancements have been viewed as invaluable tools in enhancing mathematics and geometry education. Researchers have explored the use of dynamic models, computerized programs, digital technologies, augmented reality, and virtual reality to facilitate teaching and improve students' geometric skills (Akyuz 2016; Buentello-Montoya, Lomelí-Plascencia, and Medina-Herrera 2021; İbili et al. 2020; Kristinsdóttir et al. 2020; Santos-Trigo, Moreno-Armella, and Camacho-Machín 2016).

In conclusion, addressing misconceptions, adopting innovative teaching methods, and leveraging technological advancements are crucial steps in enhancing geometry education and fostering students' mastery of geometric concepts.

**Frayer Model**

**The Emergence of the Frayer Model**

The Frayer Model is a powerful tool for helping students grasp vocabulary and concepts. Developed by Dorothy Frayer, Klausner, and Fredrich at the University of Wisconsin in 1969, this model builds upon Bruner's research on concept acquisition. Its primary goal is to support students in developing a thorough understanding and mastery of concepts. Through this approach, students engage deeply with specific terms, concepts, or phenomena, exploring their key attributes and relevant examples in detail.
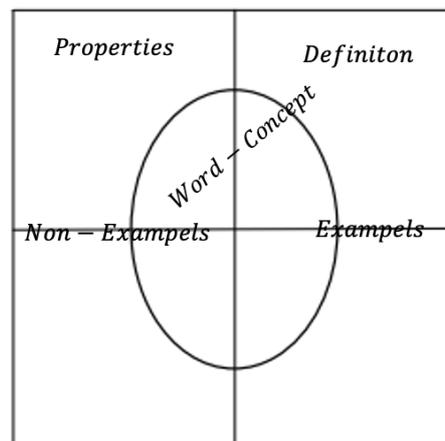


Figure 5. Frayer model

The Frayer Model is a method by which students use a graphical organizer to differentiate between topics they are learning, according to Bruun, Diaz, and Dykes (2015). The schematic for the Frayer Model by presenting the word for the target topic, describing its fundamental characteristics, and offering both indicative and non-indicative examples, each major teaching objective is specified. With this method, students may understand the subtle differences between related concepts and distinguish them clearly.

The results of this study are consistent with earlier studies that demonstrate the Frayer Model's efficacy as a strong method for enhancing conceptual understanding rather than just a straightforward vocabulary tool. The Frayer Model employs a graphical organizer that engages students in identifying the concept's name, qualities, and both indicative and non-indicative instances in order to assist them differentiate between concepts, as explained by Bruun, Diaz, and Dykes (2015). Students' capacity to recognize subtle distinctions between related concepts is facilitated by this framework. Additionally, Estacio and Martinez (2017) stress that this schematic organizer helps students understand the nuances of basic concepts, proving its worth as a crucial teaching tool for organizing and internalizing important information.

The usefulness of the Model in vocabulary education is further demonstrated by Roepke and Gallagher (2015), who demonstrate how graphical organizers methodically aid in vocabulary growth and conceptual understanding. According to Starke (2020), adding explanatory visuals strengthens this effect by graphically reiterating content and making the differences between right and wrong examples clear. Together, these studies support the present research's conclusion that the Frayer Model effectively fosters deeper, more precise understanding of geometric concepts, moving beyond surface-level recognition toward meaningful learning.

**Frayer Model's Implementing Stages**

Before implementing the Frayer Model in teaching, it is crucial to enhance its effectiveness by considering the following steps: The teacher introduces the model using relatable terms and clearly defines the expected response level, incorporating images and symbols when necessary. The teacher reviews a list of key conceptual words with students prior to reading, then proceeds with the reading itself. A chosen keyword from the reading is used to collaboratively complete the Frayer diagram. Students are provided with blank or self-created Frayer diagrams to work in pairs or small groups, each focusing on different concepts. The groups then share their diagrams, adding words, images, and symbols, or students may identify keywords in partially completed diagrams.

According to the study by Weber, Johnson, and Tripp (2013), the stages of the Frayer Model include: initial concept analysis with names and definitions, structured learning to facilitate the comprehension of the concept, and a concept acquisition stage where the tool is used to assess the effectiveness of acquisition, considering cognitive processes and learners' prior knowledge.

The Factors that Influence the Learning and Teaching Process in Frayer Model Include

Adams and Pegg (2012) pointed out that there are mainly two factors that affect the teaching and learning processes from Frayer's point of view: the internal factors which are factors related to the learner himself, such as readiness, motivation, ability to distinguish, perceive, classify, etc. While the external factors, are represented in facts not related to the learner, such as the content, the teacher, the school environment, the environment and society.

**The Advantages of Using the Frayer Model Include**

The efficacy of the Frayer Model in concept acquisition is well established through extensive research. Its adaptability and distinctive attributes make it applicable across diverse domains. Notably, the National Behavior Support (2015) highlights its value in facilitating connections between prior knowledge and new concepts, whether used before, during, or after reading. This approach enhances retention, promotes critical thinking, and supports the practical application and comprehension of interrelated concepts. Cox (2014) underscores the model's broad

applicability and advantages in concept mastery. Its key benefits include bridging prior knowledge with new concepts, offering flexibility across various learning settings, fostering critical thinking, and providing visual reference points for comparison. These attributes make the Frayer Model a versatile and effective instructional tool.

**Some Challenges Associated with Using the Frayer Model Include**

Greenwood (2002) highlights challenges hindering the successful implementation of the Frayer Model, including potential group imbalances and time constraints. Eakluas (2019) adds that learners may struggle to find suitable examples and the visual representation process can be time-intensive. Despite these issues, the model's value endures. McNeil (2015) also found significant benefits in physics concept acquisition using the dimensional Frayer Model. These studies underscore the model's importance for concept development, particularly when appropriately applied.

**Application of Frayer Model in Geometry**

Content literacy is pivotal for students' effective engagement with academic subjects like geometry. The versatile Frayer Model is widely applied across disciplines, including geometry, as a graphic organizer that helps students understand vocabulary and complex concepts (Armstrong, Ming, & Helf, 2018). This approach facilitates a precise exploration of academic language, fostering mathematical thinking skills in educational settings. Since all students are, in essence, math language learners, teachers must address linguistic challenges in mathematics, where familiar words often take on distinct meanings (Dunston & Tyminski, 2013). Implementing the Frayer Model in math education, particularly in geometry, helps correct misconceptions about terms and establishes accurate conceptual connections.

Example (1):

I can say that when passing on a new concept to be taught in geometry according to the Frayer model, we must make the student able to do the following skills:

The student should give an example of the concept if given the name of the concept. If the concept is "Circumferential angle in a circle," the student must be able to draw it as in Figure 6.
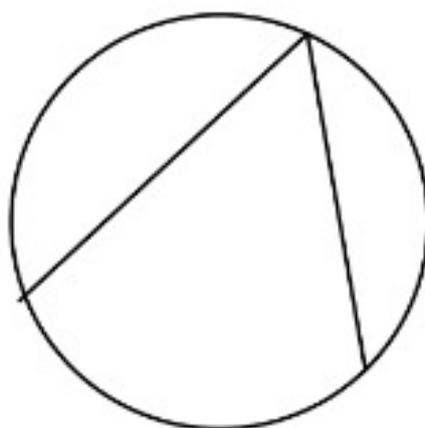


**Figure 6. Circumferential angles**

The student should mention the name of the concept, if we gave the student an example of the concept. If we gave the student a drawing as in Figure 6, he should know that the name of the concept is "a circumferential angle in a circle."

The student should give the name of the concept if we gave him a negative example of the concept. If we gave the student a drawing as in Figure 7 or as in Figure 8 or as in Figure 9, It is important for the student to understand that the concept is specifically named "circumferential angle in a circle." Additionally, the student should recognize that the angles shown in the three drawings do not qualify as circumferential angles.
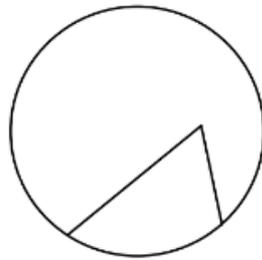
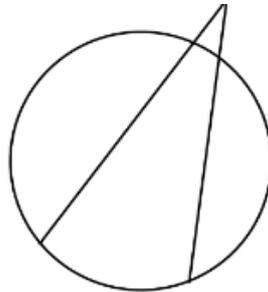Figure 7. Non circumferential angles

Figure 8. Non-circumferential angles
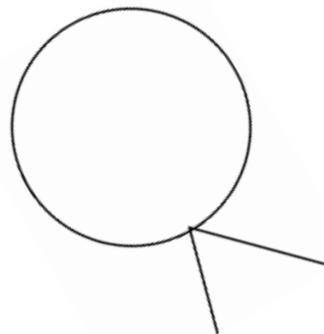
Figure 9. Non-circumferential angles

The student should be able to give an example of a declarative adjective if we gave him this characteristic. If the characteristic "an angle whose vertex is located on the circumference of a circle and its legs are two strings in a circle" is presented, the student must be able to give an example as Figure (6).

The student must write the distinctive defining characteristic if given the name of the concept. If we ask the student what are the characteristics of a "circumferential angle in a circle," he must mention two characteristics, the first is "the angle whose vertex lies on the circumference of the circle" and the second is "the angle whose legs are two strings in the circle."

He should be able to give the name of the concept if we gave to him a definition of it. So, if we ask a student what does the sentence "an angle in a circle whose vertex is on the circumference of the circle, and whose legs are two strings in the circle" mean? He should know that this is the definition of "Circumferential angle."

He should be able to write a definition of the concept if the name of the concept is given. If we ask a student to define a circumferential angle in a circle, he must know that "an angle in a circle whose vertex lies on the circumference of the circle, and whose legs are two strings in the circle."

He should be able to understand the main concept if we give to him the name of the concept. If we ask a student, "what is a circumferential angle?" he must be good at defining it and can give examples or draw it.

He should be able to give the sub-concept if we give to him the name of the concept. If the student is asked, "what are the components of a circumferential angle?" He must be able to define: a vertex on the circumference as the first component and the two legs are also strings in the circle as the second component.

He should be able to see the relationship between two concepts if we give to him their names. If we ask the student "what is the resulting construction from an angle that has a vertex on the circumference and has two legs that are strings in the circle," he has to determine it is the circumferential angle.

The student should be able to apply, to solve problems related to the concept. Which of the following angles can be circumferential angles in a circle?
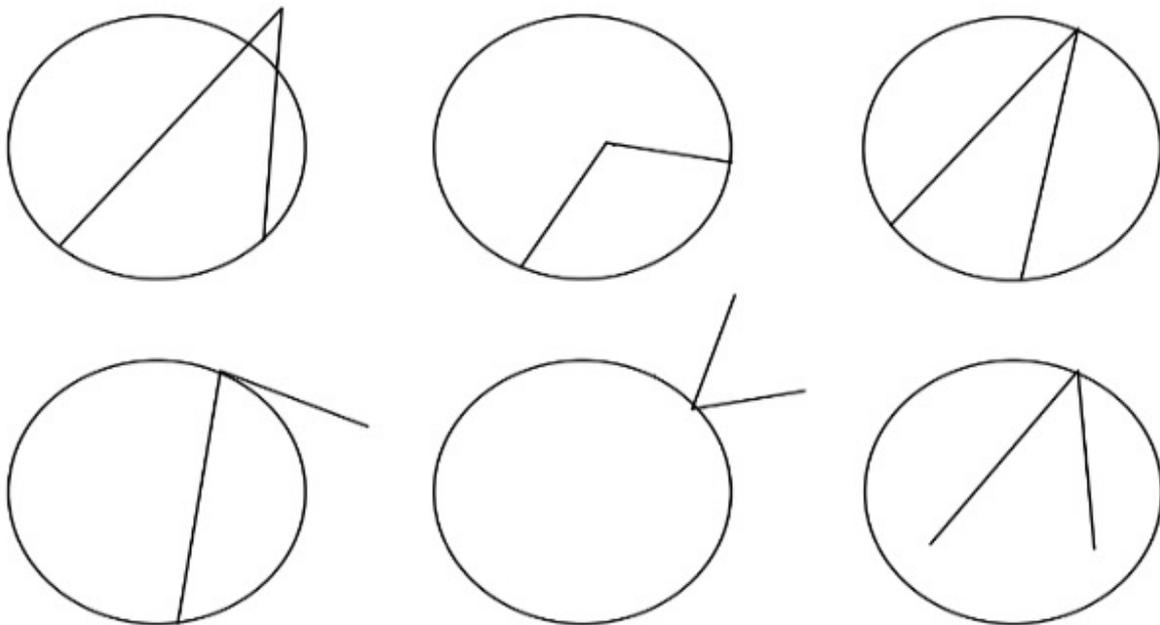


Figure 10. Which can be a circumferential angle?

The researcher acknowledges that this study was conducted in middle schools within the Israeli Arab community, a minority population. However, he believes it has potential applicability to a new target audience, particularly given its focus on "skills," which is of interest.

Notably, two Israeli researchers, Sharkia and Kohen (2021), conducted a similar study on high-achieving high school students in mathematics within the Israeli Arab sector. Their study, titled

Flipped Classroom Among Minorities in the Context of Mathematics Learning: The Israeli Case, reflects the realities of this population.

Similar studies conducted in other parts of the world, such as the United States, have examined mathematical reasoning abilities and their role in predicting future success in mathematics among various minority populations. Research by prominent scholars like Noble and Morton (2013) has presented findings that are both comparable and distinct.

**Methodology**

The research was conducted at a Galilean school comprising three ninth-grade classes: Grade 9A with 26 students, Grade 9B with 24 students, and Grade 9C with 32 students, totaling 82 participants. These classes were considered homogeneous in academic ability based on evaluations from the school principal, teachers, and students' past academic performance.

At the outset, participants were categorized into three groups corresponding to their classes, and their involvement in the study occurred in distinct stages aligned with the research procedures.

A separate random sample of thirty ninth-grade students who were not part of the main study participated in a pilot test prior to the main study. The researcher and geometry teachers worked together to create this prototype accomplishment test, which evaluated students' comprehension of geometric ideas from the "Angles and Arcs in the Circle" lesson. There were fifteen multiple-choice vocabulary items on the test, each with four possible answers and one right response.

Analysis of the pilot test results (see Table 1) revealed a general weakness in students' grasp of geometric concepts across all three ninth-grade classes. Additionally, the data confirmed comparable achievement levels among the three grades prior to the intervention.

**Table 1: Results of the Achievement Detection Test in Geometric Concepts of the Galilee School**

| Concepts | Failed | Successful | Percentage |
|---|---|---|---|
| Geometric Concepts | 16 | 14 | 53.3% |

Therefore, it was possible to choose the ninth A, (with 26th students), as an experimental group and the ninth B (with 24th students) as a control group. This is to prevent the transfer of the impact of learning from the experimental group to the control group as well as to facilitate the school for arrangements to conduct research, and so the research group consists of (50) students.

**Research Tools**

To conduct the research, the researcher prepared along with the geometry teachers at the selected school the following:

a list of geometric concepts included in the experimental unit.

An activity booklet for the student to practice geometric concepts in the experimental unit.

A teacher's guide explaining steps to implement the Frayer Model in teaching the experimental unit.

An achievement test to measure the geometric concepts included in the experimental unit to be taught according to the three levels of Bloom (remembering - understanding - applying).

Research Procedure

The researcher adopted the experimental design based on the control and experimental groups in the application "pre" and "post" measurement to assess the impact of using the Frayer Model in developing geometric concepts among middle school students according to the three levels of Bloom (remembering-understanding-application), (See Table 2).

**Table 2: Experimental Design Scheme for Research**

| Group | Compatibility | Independent Variable | Dependent Variable |
|---|---|---|---|
| Control (n=24) | Previous Achievement | Frayer Model | geometric concepts |

One of the requirements of the current research is to prepare a test to measure geometric concepts in the experimental unit to be taught about the educational unit of the circle. In Table 3 specifications were developed for this test and the relative weights and the number of test parts were determined, as the test contained (40) items of the type of multiple choice for each item there are (4) alternatives, of which only one is correct; By creating exam questions and learning activities that were in line with each cognitive domain, the three levels of Bloom's taxonomy—memory, comprehension, and application—were made operational. Items that tested students' ability to accurately recall geometric terminology and definitions were employed to gauge their memory. Students were asked to categorize examples and non-examples, evaluate diagrams, and explain topics in their own words in order to thoroughly examine their level of overall comprehension. Applications included assignments and tasks that required students to address issues in novel contexts using geometric concepts, such as evaluating figures or applying theorems to actual situations. From fundamental recollection to applied reasoning, this tiered framework made sure that pupils' conceptual development was evaluated gradually. (See Table 3).

**Table 3: Table of Specifications for Testing the Geometric Concepts Included in the Unit**

| Item | Elements of Unit | Levels of Achievement | | | Total |
| | | Remembering | Understanding | Application | |
|---|---|---|---|---|---|
| 1 | Central Angles and Measurement of Arcs | 4 | 5 | 4 | 13 |
| 2 | Relationship between Central and Circumferential Angles Shared by Arc | 1 | 2 | 1 | 4 |
| 3 | Circumferential Angles Drawn on the Same Arc | - | - | 3 | 3 |
| 4 | Circular Quadrilateral | 1 | 1 | 2 | 4 |

| | | | | | |
|---|---|---|---|---|---|
| 5 | Characteristics of the Circular Quadrilateral Relationship | - | - | 4 | 4 |
| 6 | between Tangents to a Circle | 2 | 2 | 2 | 6 |
| 7 | Tangent Angle | 2 | 3 | 1 | 6 |
| Total | | 10 | 13 | 17 | 40 |

The test was administered to raters, including middle school teachers and geometry guides, with necessary adjustments made. Statistical significance was assessed using a two-tailed analysis. To prepare for statistical research analysis, an exploratory test was conducted on a separate sample of 32 ninth-grade students from class C in the same school. Their responses were evaluated, and 27% of students were selected from both the upper and lower halves of the distribution to compute the difficulty coefficient, which ranged from 0.46 to 0.71. Discriminant validity for each item ranged from 0.25 to 0.563, confirming the test's suitability. Test stability, assessed using Cronbach's alpha, yielded a coefficient of 0.75, indicating high reliability (Table 4).

**Table 4: Calculation of the Stability Coefficient of the Exploratory Test Geometry Concepts**

| The Number of Test questions (n) | Population variance | Arithmetic mean | sample variance | Coefficient reliability $\alpha$ |
|---|---|---|---|---|
| 40 | 9.168 | 25.469 | 34.515 | 0.7532 |

**Results and Discussion**

Before training, a geometric concepts accomplishment test was conducted for both the experimental and control groups to make sure they understood each other. The findings of the t-test, which was used to determine if the differences between the experimental and control groups in the pre-application of the geometric concepts test were significant, are shown in Table 5.

Prior to the t-test analysis, homogeneity of variances was assessed using the F-test (F-percentage calculation). The calculated F-value was 3.077, which exceeds the tabular F-value of 2.68 (df = 25, 23, $\alpha$ = 0.01). Therefore, the assumption of homogeneity of variances is rejected, indicating heterogeneity between the groups.

**Table 5: T-test Results**

| The group | n | mean | standard deviation | t value calculated | tabular | significance 0.01 |
|---|---|---|---|---|---|---|
| control | 24 | 16.875 | 1.766 | 1.971 | 2.406 | Not significant |
| experimental | 26 | 17.923 | 5.434 | | | |

From Table 5 we note that the value of (t) is not a function at the level of (0.01), which indicates the adequacy of the two study groups in the pre-application of testing geometric concepts before the start of the experiment.

Teaching the Unit "Angles and Arcs in a Circle" for the Two Study Groups

In order to make sure that every instructional step was in line with pedagogical theory and the intended learning outcomes, the researcher helped the teacher apply the Frayer Model in the experimental group's (Grade 9A) classroom. Based on Vygotsky's social constructivist philosophy, the exercises emphasized scaffolding and directed interaction while students worked with novel geometric ideas. In order to successfully operationalize the Frayer Model, the instructor used a specific geometric word to introduce the model, clearly described its elements, and used think-aloud techniques to model the cognitive process. In accordance with Bloom's taxonomy levels of comprehension and application, this method was created to foster metacognitive awareness and enhance conceptual understanding. In order to improve their capacity to distinguish between related concepts, students were also assisted in creating examples and counterexamples that were backed up by symbols and visual representations. These exercises were designed to develop longer-lasting conceptual knowledge and higher-order thinking skills in addition to word acquisition.

Reviewing the List of Pre-Prepared Conceptual Keywords with The Students

Moving to teach the term according to its material in the textbook or the material prepared by the researcher with the teacher for this purpose.

The teacher and researcher highlight the main term to be taught and help the students complete the Frayer Model with keywords according to what has been taught.

Distribute blank copies of the Frayer Model to students or they can create the copies themselves.

Students are then assigned to implement the strategy in pairs or small groups using the key concepts and terms contained in the topic taught, and different conceptual words can be given to each group.

The groups share their completed performance with each other, after which students can add additional words, pictures, and symbols until the four categories are adequately complete.

Also, on some occasions, students were given a complete chart except for the key word which they had to find out. The researcher and the teacher also passed all the concepts in the educational unit "angles and arcs in the circle" similar to example (1), which had been mentioned earlier.

At the same time, the ninth teacher explained to the control group according to the traditional method.

The experiment lasted a full month, which was October, 2022. Each week, students were given five geometric classes to cover the contents of the unit, and during the month they were given 20 lessons. After that, the research test was applied and its results were monitored in preparation for statistical processing.

The paradigm accomplishes five major learning goals, all of which are meant to promote idea mastery. It helps students clarify the idea in their own words, pinpoint its key features, and give examples that are both suggestive and non-indicative. This material is arranged methodically in a table with four sections surrounding the main idea. This visual aid improves students' understanding of the idea, as seen in Figure 5 (Reed et al., 2019).

To answer the research question, the researcher analyzed the test results for each of the two groups where the results were as shown in Table 6.

**Table 6: The Results of the "t" test to Detect the Significance of the Differences Between the Experimental and Control Groups in the Dimensional Application of the Geometric Concepts Test**

| The group | n | arithmetic mean | standard deviation | t value calculated | t value tabular | significance 0.01 | Eta-squared �2 $\eta 2$ |
|---|---|---|---|---|---|---|---|
| Control | 24 | 17.166 | 2.927 | 11.303 | 2.406 | Significant | 0.727 |
| Experimental | 26 | 28.115 | 21.226 | | | | High |

The Frayer Model has a significant impact on students' comprehension of geometric ideas, as evidenced by the study's significant effect (t-value of 11.303 and ETA-squared of 0.72). This outcome can be explained by a number of educational and cognitive processes. By integrating verbal and visual representations of information, the Frayer Model cognitively supports dual coding theory, which improves memory retrieval and retention. The paradigm encourages deep processing and conceptual differentiation by asking students to describe a topic, enumerate its key characteristics, and provide instances as well as non-examples. By actively involving students in the process of creating meaning and providing scaffolding for their comprehension through organized inquiry, it adheres to constructivist pedagogical concepts.

Because students must examine their own thinking in order to discern between accurate and inaccurate representations, the paradigm also encourages metacognition. These components work together to produce a rich learning environment that goes beyond simple memory to integrated and transferable comprehension, which explains the experimental group's notable learning improvements.

The significance of these results extends beyond mere numerical validation; they underscore the pedagogical impact of structured vocabulary instruction in mathematics. The findings align closely with Aveyard's (2018) study, which emphasized the Frayer Model's efficacy in teaching technical vocabulary to ninth-grade students. This structured approach to learning terminology not only enriched students' comprehension but also fostered their ability to engage with complex mathematical concepts, ultimately improving their overall achievement. Similarly, Bruun, Diaz, and Dykes (2015) arrived at parallel conclusions, demonstrating that students across various proficiency levels benefited from the model's emphasis on conceptual clarity. Their research reaffirmed the crucial role of mathematical vocabulary in problem-solving, further solidifying the case for integrating the Frayer Model into mathematics instruction.

A key factor contributing to the success of this study is the dynamic interaction facilitated by the Frayer Model. The experimental group exhibited a heightened level of engagement, as discussions between students and teachers fostered deeper interpretations and reflections on geometrical concepts. This interactive process echoes Dunston and Tyminski's (2013) argument that effective vocabulary instruction enhances adolescents' abstract thinking and problem-solving abilities, extending beyond routine calculations to more complex mathematical reasoning. By enabling students to articulate their understanding, the Frayer Model nurtures not just retention but also the practical application of knowledge.

Additionally, the study's success can be attributed to the unique sociocultural context in which it was conducted. The students in this Galilee-based community approached the Frayer Model with curiosity and optimism, recognizing its potential advantages even before the intervention began.

 (If no pre-intervention data exists):

While there is no formal evidence to confirm students' initial attitudes toward the Frayer Model, their active engagement during instruction suggested openness to the approach and a willingness to explore new methods for learning geometric concepts.

 (If some observational or anecdotal evidence exists):

Based on preliminary classroom observations and informal student remarks, learners in the Galilee-based community appeared to approach the Frayer Model with curiosity and a positive attitude, which may have contributed to their engagement during the intervention.

 This anticipatory enthusiasm likely amplified their motivation, reinforcing the model's effectiveness. Hale's (2017) research similarly pointed to the role of intellectual stimulation in enhancing learning outcomes, illustrating how non-traditional instructional methods can captivate students and facilitate deeper comprehension of abstract concepts. Aryanti (2017) further corroborates this perspective, emphasizing that innovative approaches, such as the Frayer Model, provide a framework that strengthens students' cognitive and communicative abilities in mathematical contexts.

Ultimately, the findings of this study reaffirm the indispensable role of structured vocabulary instruction in mathematics education. By bridging the gap between language and conceptual understanding, the Frayer Model empowers students to navigate complex mathematical frameworks with confidence. Given the compelling evidence of its impact, future research should explore its application in other mathematical domains and grade levels to assess its broader effectiveness. Educators and curriculum developers are encouraged to incorporate the Frayer Model into their instructional strategies, recognizing its potential to transform students' learning experiences and foster a deeper appreciation for mathematics.

**Recommendations**

Based on the findings of this study, which reiterated the positive impact of utilizing the Frayer Model in teaching geometric concepts among ninth-grade students in the Galilee region of Israel, numerous recommendations emerge for various stakeholders, beneficiaries, and research communities in the field of mathematics education.

**For Educators and Schools**

It's crucial to situate the Frayer Model within more general curriculum objectives rather than just providing helpful advice for specific educators. Its incorporation into the mathematics curriculum specifically serves important goals including encouraging conceptual understanding, cultivating mathematical language, and improving students' communication and reasoning abilities. The strategy is in line with curricular requirements that prioritize in-depth learning and the application of knowledge in a variety of contexts by organizing education around exact terminology and conceptual differentiation. In addition to improving students' understanding of geometric concepts, this methodical approach helps them acquire

higher-order thinking skills that are specified in national and international mathematics frameworks.

**For Students**

The integration of the Frayer Model in mathematics classrooms maintains substantial pros for students, as it propagates a more structured and analytical avenue to learning geometric jargon. The results of this study indicate that students who actively and effectively engaged with this model demonstrated a clearer conceptual comprehension of geometric terms related to the study unit on the circle. On a broader respect, their mathematical discourse meliorated, allowing them to generate their reasoning with notably higher precision. To further capitalize on these benefits, students should have access to well-constructed worksheets and activity handouts specifically designed to reinforce their awareness of geometric concepts. Such resources, when carefully and systematically aligned with the Frayer Model, will provide students with opportunities for repeated practice, monitored reflection, and application of their actual knowledge in numerous related settings and contexts

**For Curriculum Developers**

The results of this study shed the light on the necessity of reconsidering how geometric concepts are presented and taught within middle school mathematics curricula. Curriculum developers should explore ways to actively and effectively incorporate the Frayer Model into instructional materials, ensuring that conceptual development is prioritized alongside procedural fluency. The structured organization of content, with explicit emphasis on defining, visualizing, and applying mathematical terms, can massively improve students' capacity to navigate challenging geometric problems. Furthermore, the development of teacher's guides that provide step-by-step methodologies for implementing the Frayer Model could serve as a valuable asset in standardizing best practices across different educational institutions, settings, and environments.

**For Researchers**

This work opens avenues for further and more targeted research into the efficacy of (ALS) active learning strategies in mathematics education. Future studies could possibly tackle the longitudinal effects of the Frayer Model on students' retention of geometric concepts, their capacity to transfer knowledge through various mathematical disciplines, and the model's applicability to other domains of mathematics, namely Algebra and Calculus. Comparative studies evaluating the effectiveness of the Frayer Model against other (CLSs) conceptual learning strategies could come up with more thorough insights into optimizing mathematics instruction. Additionally, researchers may scrutinize how factors such as students' cognitive patters, language proficiency, numeracy affect the effectiveness of this instructional approach at large.

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**Conflict of Interest**

The authors declare that there is no conflict of interest.

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